STEADY STATE FLOW OF DETONATING GAS AROUND A CONE

(USTANOVIVSHEESIA OBTEKANIE KONUSA POTOKOM DETONIRUIUSHCHEGO GAZA)

PMM Vol.23, No.1, 1959, pp.182-186

S.S. KVASHNINA and G.G. CHERNYI
(Moscow)

(Received 11 November 1958)

There is much theoretical and experimental work concerning detonating combustion of gas mixtures. Owing to the high velocity of propagation of detonating waves (order of several km/sec), the experimental studies covered only the unsteady motion of a gas due to the propagation of detonation in an undisturbed medium. Theoretical solutions also have been concerned mainly with the propagation of planar, cylindrical and spherical detonation waves in an undisturbed gas of a constant or variable density [1-4].

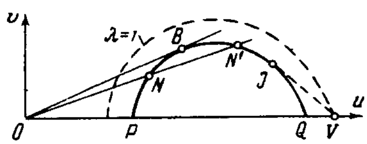

Fig. 1.

Recently, in connection with the problem of burning fuel in supersonic flow [5-6], there has been a renewed interest in the study of detonative waves in steady-state flow, and reports on flow around a body of a gas capable of detonation are beginning to appear.

In reference [7] is derived an equation which, for a given velocity and ratio of stagnation temperatures behind and ahead of the detonative wave, relates the velocity components behing the wave (equation of detonative polar). Segment *PS* of a detonative polar (Fig. 1) corresponds to strong (or over-compressed) detonative waves where the normal component of velocity behind the wave is subsonic. Segment *JQ* corresponds to a weak (or under-compressed) detonative wave where the normal component of velocity behind the wave is supersonic. Point *J*, dividing these two types of detonation (point *J* is obtained by drawing a tangent to the polar from point *V*, free stream velocity) corresponds to the Chapman-Jouguet detonation where the normal component of velocity behind the wave is equal to the local speed of sound. Note that under-compressed waves of detonation do not exist [1] under usual conditions.

In reference [8] the problem of the flow of detonating gas around a wedge is solved. If angle θ, formed by the side of the wedge and the direction of flow, is larger than the angle between the direction of flow and the tangent OB (Fig. 1,), then flow with an attached detonative wave is impossible and a detached wave is formed ahead of the wedge. At smaller values of θ a line formed by the side of the wedge has two points of intersection with the polar (points N and N', Fig. 1).

As in the case of a simple shock wave near point N', velocity behind the detonative wave is supersonic (excluding a small region in the vicinity of point B), while near point N it is subsonic. As the wedge angle θ decreases until points N' and J coincide, i.e. $\theta = \theta_j$, the Chapman-Jouguet detonation occurs, the velocity component of the burnt gas normal to the wave being equal to the speed of sound. Then the wave coincides with the straight-line characteristics of supersonic flow behind it. If the angle is still further reduced, the detonative wave remains as the Chapman-Jouguet detonation. However, from the straight-line characteristic which coincides with the detonative wave there originates the Prandtl-Meyer flow in which the flow turns from angle θ_j in direction $\theta < \theta_j$ along the side of the wedge. In the limiting case when $\theta = 0$, flow in the rarefied wave again assumes its original direction. In this case the solution corresponds to the propagation of a detonative wave from a straight line source of ignition perpendicular to the flow. An example of such a flow is shown in Fig. 2.

It is experimentally easier to study the flow of stationary detonative waves around bodies of revolution than around airfoils. The simplest case of a flow around a body of revolution - a symmetric flow around a circular cone - is therefore examined here.

This can be done by using the well-known theory of axisymmetric flow of a gas [9, 10]. According to this theory, the equation relating velocity component u in the direction of the axis of symmetry and v-normal to it, is

$$vv'' = 1 + v'^2 - \frac{(u + vv')^2}{a^2} \qquad (1)$$

where the primes designate differentiation with respect to u, and a is the speed of sound determined from

$$a^2 = \frac{\gamma + 1}{2}\lambda a_*^2 - \frac{\gamma - 1}{2}\left(u^2 + v^2\right)$$

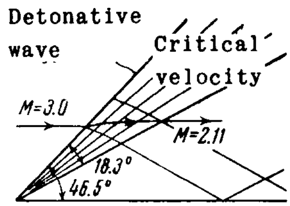

Fig. 2.

(γ is the ratio of specific heats, a_* the critical velocity ahead of a detonative wave, λ the ratio of stagnation temperatures behind and ahead of the wave). The relationship between the planes uv and xy is given by

$$\frac{y}{x} = -\frac{1}{v'}$$

i.e. when the direction of the x and u axes is identical, the normal to the integral curve $v(u)$ of equation (1) is parallel to the corresponding line in the plane xy.

In the problem of the flow of a detonating gas around a cone, the integral curve of equation (1) must begin on a polar of detonation, the equation of which [8] is

$$v^2 = (V - u)\frac{Vu(V-u) - (\lambda V - u)a_*^2}{[2/(\gamma+1)]V^2 - Vu + a_*^2}$$

(V is the velocity of the oncoming flow). According to the above relationship between the planes uv and xy, the integral curve at the initial point must be directed along the straight line (Fig. 3) which connects this point with point V. The integral curve must end at the point satisfying

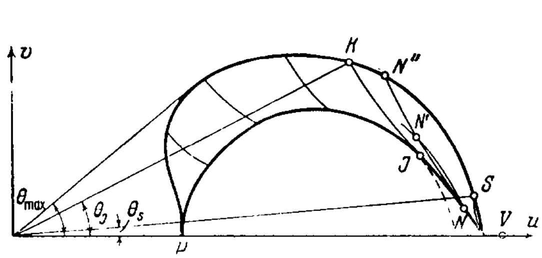

Fig. 3.

$$u + vv' = 0$$

according to the boundary condition on the surface of the cone. The condition of a unique solution in the plane xy demands that, while moving along the integral curve, the normal to the curve must rotate monotonically clockwise.

Equation (1) can be replaced by the geometric relation [9]:

$$R = -\frac{N}{1 - U^2 / a^2}$$

$$\left(R = -\frac{(1+v'^2)^{3/2}}{v''},\ N = v(1+v'^2)^{1/2},\ U = \frac{u + vv'}{(1+v'^2)^{1/2}}\right)$$

Here R is the radius of curvature of the integral curve, N is the absolute value of a segment of the normal to the integral curve between the curve and axis u, U is the velocity component tangent to the direction of the integral curve at a given point, i.e. velocity component v_n is normal to the corresponding line (Fig. 4) in the plane xy.

Since the portion of the detonative polar to the left of J corresponds to the over-compressed detonating wave, $v_n < a$, the integral curves describing the flow behind the over-compressed detonating waves are initially convex toward the u axis.

In accordance with the said fact about the normal curvature of the solution, these curves emerge from the initial point to the left and correspond to the conical compressed flow, analogous to the well-known cases of the flow through a shock wave around a cone. The ends of the integral curves, corresponding to the surface of the cone (the normal to the integral curves at these points must pass through the origin of the coordinates in order to satisfy the required boundary condition $v_n = 0$), form segment PK in Fig. 3, analogous to the "apple" curve when $\lambda = 1$.

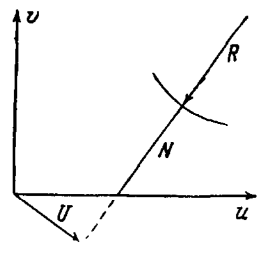

Fig. 4.

At point J, which corresponds to the Chapman-Jouguet detonation, $v_n = a$; therefore the radius of curvature of the integral curve at this point is infinite.

Examining the mutual shape of the integral curve and the curves $v_n = a$ (which, in the case of a perfect gas with a constant specific heat, are epicycloids), it is easy to show that the integral curve has an inflection point at J. Therefore it is possible to move from J along the integral curve not only to the left but also the the right while keeping rotation of the normal clockwise. To the latter case corresponds the beginning of rarefied flow behind the detonative wave, in which the direction of gas flow approaches the direction of the axis of symmetry. In this rarefied flow on each line emerging from the apex of the cone, the velocity component normal to the line is supersonic, and therefore each such line can be replaced by a conical shock wave.

Let us examine a certain point N on the integral curve which describes a rarefied flow (Fig. 3) and draw through this point a shock polar, the initial velocity of which is ON. Point N' in the plane uv corresponding to velocity behind the shock wave must be the intersection of the shock polar with the tangent to the integral curve at N.

From the condition of continuity in the plane xy it follows that the direction of the discontinuity, determined as the direction of the normal to the secant NN', must coincide with the direction of the normal to the integral curve at N. Since $v_n < a$ at N', the integral curve corresponding to the flow behind the shock wave is concave toward the u axis and must initiate at N' in the direction of NN'. This curve describes a compressive flow and extends (as do the curves emerging from the points on the segment PJ of the detonative polar) to N'', where $v_n = 0$. The intensity of the shock wave while approaching J reduces to zero, since at this point the shock polar is tangent to the integral curve. This is so because the shock polar and the epicycloid $v_n = a$ at this point have a common tangent to the integral curve. For the same reason, the intensity of the shock wave tends to zero when N approaches point L located on the u axis. The locus of the points N'', corresponding to all positions of N on the segment JL of the integral curve, forms segment KS of the "apple" curve.

Beyond point L the flow can be extended by various means. From this point, which is a singular point of the differential equation (1) (at the same angle as the integral curve JL), emerges a family of integral curves of various curvatures [9]. From the condition that the solution in the plane xy does not change sign, it follows that for the extension of the solution corresponding to the integral curve JL, those integral curves which at this point have curvature of an opposite sign than the curve JL, should be utilized. These integral curves describe a compressed flow continually adjacent to the rarefied flow JL. The ends of the integral curves which satisfy the condition $v_n = 0$ form the remaining part of the "apple" curve.

The possibility of the continuity of the rarefied flow described by the integral curve JL with various compressed flows can be justified by the fact that the boundary line of the rarefied flow corresponding to the point L is a characteristic.

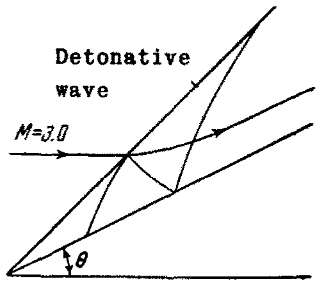

Fig. 5.

The preceding analysis indicates that the flow of detonating gas around a cone may have several aspects. For each cone angle smaller than a certain limiting value of θ_{max}, depending upon Mach number λ and γ, there can be two regimes of flow around the cone with an attached wave of detonation. Evidently, as in the case of an inert gas, in a free-stream flow of a detonating mixture around a cone, there will be a regime which corresponds to a weaker detonative wave. If the cone angle is less than θ_{max} but more than or equal to θ_j (Fig. 3), then between the detonating wave and the surface of the cone a continuous compressive flow (at $\theta > \theta_j$ the wave of detonation will be over-compressed) takes place. This case is quite analogous to the well-known flow of an inert gas around a cone with an attached shock wave. Since the velocity component normal to the rays is subsonic in this flow, the characteristics outgoing from the surface of the cone or from the Mach line (if it exists) touch the wave of detonation, and when the cone angle is changed the angle of the conical wave of detonation also changes. The mechanism of the flow is shown in Fig. 5.

When the cone angle is less than θ_j, between the wave of detonation, which remains unchanged and corresponds to the Chapman-Jouguet detonation, and the compressed flow near the surface of the cone there originates a rarefied conical zone bounded by a shock wave. As the cone angle decreases the width of the rarefied zone grows, while the intensity of the shock wave first increases and then begins to decrease. When $\theta = \theta_s$ the width of the rarefied zone is maximum, and the shock wave which bounds it degenerates into a characteristic. The flow on this characteristic is along the symmetry axis of the flow and changes its direction continuously in the compressed wave until the required direction is reached. When the angle θ is decreased still further, the intensity of the compression wave reduces and when the cone angle is zero it disappears entirely. Then, behind the conical zone of rarefaction, the flow advances along the axis of symmetry. This limiting case corresponds to the propagation of a detonative wave from a point source of ignition and also describes the flow of a detonating gas around an arbitrary finite body (including a cone when

$\theta > \theta_{max}$) at a great distance from the body.

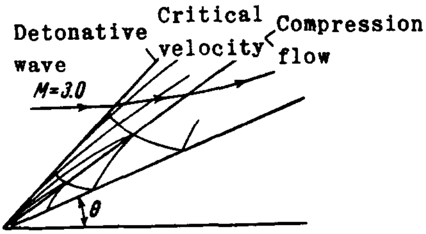

Fig. 6.

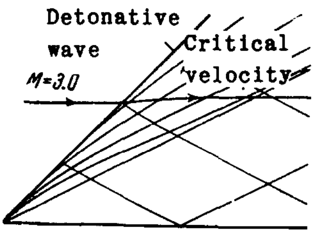

Fig. 7.

Since in the conical wave of rarefaction $v_n > a$ disturbances coming from the surface of the cone along the characteristics cannot penetrate this region or influence the location of a detonative wave.

Flows with a rarefaction wave with and without a shock are shown in Figs. 6 and 7 respectively.

The fact that, as against the case of the rarefaction wave in a planar flow, inside the conical zone of rarefaction the formation of conical shock is possible, is connected with the peculiar behavior of the characteristics in the conical wave of rarefaction (compare Figs. 2 and 7).

The motion of a cone in a detonating gas in the presence of a shock behind the wave presents an interesting example of a flow in which there are two gaps between particles moving in the same direction: the detonation wave propagating so that the particles of gas behind it have a normal velocity component equal to the speed of sound, and following it the shock wave which propagates supersonically with respect to the gas particles ahead of it.

In conclusion let us note that when the velocity of the detonating mixture is much higher than velocity of the propagation of the detonative wave, the problem of the flow around a cone can be reduced approximately to the problem of the expansion caused by a cylindrical piston moving with constant speed in a detonating gas [11].

BIBLIOGRAPHY

1. Zel'dovich, Ia. B., Kompaneets, A.S., *Teoria detonatsii* (*Theory of Detonation*). Gostekhizdat, M., 1955.

2. Bam-Zelikovich, G.M. Raspad proisvol'nogo razryva v goriuchei smesi (Decay of an Arbitrary Gap in a Combustive Mixture). *Sb. Statei Teoreticheskaia Gidromekhanika* No. 4, Str. 112-141, Oborongiz, M., 1949.

3. Sedov, L.I., O dvizhenii gaza pri zvezdnykh vspyshkakh (On Movement of gas in presence of ignition point source). *Dokl. AN SSSR* Vol. 111, No. 4, 1956.

4. Iavorskaia, I.M., Reshenie nekotorykh zadach o detonatsii v srede s peremennoi plotnost'iu (Solution of certain problems on detonation in a medium of variable density). *Dokl. AN SSSR* Vol. 111, No. 4, 1956.

5. Dunlap, R., Brehm, R.L. and Nicolls, J.A., A preliminary study of the application of steady-state detonative combustion to a reaction engine. *Jet Propulsion,* 28, 7, 1958.

6. Roy, M., *Propulsion supersonique par turbo-réacteurs et par stato-réacteurs.* Préprint, I Intern. Congr. Aéron. Sciences, Madrid, Sept., 1958.

7. Siestrunck, R., Fabri, J. and LeGrives, E., *Some properties of stationary detonation waves.* IV Symp. on Combustion, Baltimore, 1953.

8. Rutkowski, J. and Nicholls, J.A., Considerations for the attainment of a standing detonation wave. *Proc. Gas Dynamics Symp. on Aero-thermochemistry,* Evanston, Ill., 1956.

9. Busemann, A., Die achsensymmetrische kegelige Überschallströmung, *Luftfahrtforsch.*, 19, 4, 1942.

10. Nikol'skii, A.A., Konicheskie osesimmetricheskie sverkhzvukovyie gazovye techenia razriazhenia (Conical axi-symmetrical supersonic flow of rarefied gas). *Sb. teoreticheskikh rabot po aerodinamike* Str. 43-55 Oborongiz, M., 1957.

11. Sedov, L.I., *Metody podobia i razmernosti v mechanike (Methods of Similarity and Dimensionality in Mechanics).* Gostekhizdat, M., 1957.